

## EAST Search History

Ref #	Hits	Search Query	DBs	Default Operator	Plurals	Time Stamp
L3	2	"5247468".pn.	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/22 16:11
S1	0	"integrated circuit technolog\$" "model\$" "simulat" "risk" "cost" "analysis"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 15:33
S2	0	"integrated circuit" "model\$" "simulat" "risk" "cost" "analysis"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 15:48
S3	1	"integrated circuit" "model\$" "simulat" "analysis"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 15:50
S4	64247	"integrated circuit" ("model\$" OR "simulat\$")	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 15:51
S5	4590	"integrated circuit" ("model\$" OR "simulat\$") "cost" "risk"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 15:51
S6	1	"integrated circuit" ("model\$" OR "simulat\$") "cost" "risk" "analy\$"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 15:53
S7	1	"integrated circuit" "cost" "risk" "analy\$"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 16:24

## EAST Search History

S8	196	"integrated circuit" "cost" "risk" "anal\$"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 16:26
S9	4654	"integrated circuit" "cost" "risk" "analysis"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 16:35
S10	3181	"integrated circuit" "cost" "risk" "analysis" (model\$ OR simulat\$)	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/13 16:35
S11	17667	"integrated circuit" SAME ("cost" or "risk")	USPAT	OR	OFF	2006/06/14 07:41
S13	2157	"integrated circuit" SAME ("cost" or "risk") SAME "method"	USPAT	OR	OFF	2006/06/14 07:43
S14	119	"integrated circuit" SAME ("cost" or "risk") SAME "model\$"	USPAT	OR	OFF	2006/06/14 08:28
S15	127	integrated circuit.TI. ("cost" or "risk") SAME "model\$"	USPAT	AND	OFF	2006/06/14 07:44
S16	486	integrated circuit.AB. ("cost" or "risk") SAME "model\$"	USPAT	AND	OFF	2006/06/14 08:00
S17	211	("integrated circuit" OR "IC") SAME ("cost" or "risk") SAME "model\$"	USPAT	OR	OFF	2006/06/14 08:00
S23	7	("cost analysis".TI. OR "risk analysis".TI.) AND ("integrated circuit" OR "IC")	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:13
S26	16	("cost analysis".TI. OR "risk analysis".TI.) AND "comparing"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:12
S27	15	("cost analysis".TI. OR "risk analysis".TI.) AND "technology"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:07

## EAST Search History

S28	6	("cost analysis".TI. OR "risk analysis".TI.) AND "comparing" AND "technology"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:08
S29	3	("analysis".TI.) AND ("risk".TI.) AND ("cost".TI.)	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:12
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S31	12	("cost analysis".TI. OR "risk analysis".TI.) AND "graph\$"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:14
S32	1	("cost analysis software" OR "risk analysis software") SAME ("input" OR "output")	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:33
S33	4	(compar\$6 ADJ technolog\$ ) AND ("cost analysis" OR "risk analysis")	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:36
S34	0	(compar\$6 ADJ technolog\$ ) AND ("cost ADJ analysis" OR "risk ADJ analysis")	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:36
S35	0	(compare ADJ technologies ) AND ("cost ADJ analysis" OR "risk ADJ analysis")	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:36

## EAST Search History

S36	0	(compar\$3 ADJ technolog\$) AND ("cost ADJ analysis" OR "risk ADJ analysis")	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:37
S37	0	("cost ADJ analysis" AND "risk ADJ analysis") AND "integrated ADJ circuit"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:51
S39	1042	(cost NEAR5 risk) ((integrated ADJ circuit) OR IC)	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:54
S40	645	(cost NEAR5 risk) ((integrated ADJ circuit) OR IC) output input	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 08:57
S41	915	(cost NEAR5 risk) computer software variable technolog\$ output input	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 09:00
S42	826	(cost NEAR5 risk) computer software variable technolog\$ output input compar\$7	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 09:00
S43	0	"integrated ADJ circuit"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 15:27
S44	0	"integrated ADJ circuit"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 15:27

## EAST Search History

S45	613309	"integrated circuit"	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 15:27
S46	188417	S45 AND technology	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 15:27
S47	8891	S46 AND cost AND risk	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 15:28
S48	274	S47 AND (technology SAME cost SAME risk)	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/14 15:29
S49	2	"6795800".pn.	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/15 14:29
S54	17	"5247468" Henrichs	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/22 16:09
S55	9	compare same risk same cost same circuit	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/19 13:41
S56	223	compare same risk same cost	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/19 13:41

## EAST Search History

S57	18581	input value slid\$4 (real ADJ time)	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/21 08:41
S58	102	input SAME value SAME slid\$4 SAME (real ADJ time)	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/21 08:42
S59	13	(input SAME value SAME slid\$4 SAME (real ADJ time)) and integrated ADJ circuit	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/21 08:51
S60	13	(input SAME value SAME slid\$4 SAME (real ADJ time)) and (graph or display or chart) and integrated ADJ circuit	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/21 08:55
S61	11	(input SAME value SAME slid\$4 SAME (real ADJ time)) and (graph or display or chart) and (dynamic\$4 or instant\$6 or immediat\$4) and integrated ADJ circuit	US-PGPUB; USPAT; USOCR; EPO; JPO; DERWENT; IBM_TDB	AND	ON	2006/06/21 12:55

- MATHEMATICS AND COMPUTER EDUCATION -

## USING APPLETS IN TEACHING MATHEMATICS

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### INTRODUCTION

During the last several years the Department of Mathematical Sciences at the United States Military Academy has significantly increased the use of technology in the classroom. Currently students have and use the TI-89 calculator as well as *MathCad* 2000. These technology advances can increase a student's ability to learn objectives and concepts presented in Discrete Dynamical Systems and Introduction to Calculus, and Introduction to Calculus and Differential Equations, the first two required freshman level mathematics courses. However, these tools have an associated cost for the majority of students who must learn the hardware and/or software while trying to grasp new course material. Informal surveys of students repeatedly indicate this to be perceived as detrimental to learning. To minimize this negative effect on the learning process I have found it beneficial to use applets ("a program designed to be executed from within another application." [2, p. 1]) in classroom demonstrations, and to allow students to use them in completing suggested problems. This does not free them from the responsibility to learn the hardware/software; however, it does equip students with tools to immediately focus on concepts and learning objectives.

### BACKGROUND

*Mathcad* and the TI-89 brought a wealth of power and benefits to both the instructor and student. For the instructor, both enhanced classroom demonstrations. *MathCad* made it much easier to graphically show the effects of modifying parameters of an equation. Attempting to accomplish the same with a blackboard and/or transparencies is a futile effort. We found much the same was true for demonstrations with the TI-89. However, the TI offered a bigger plus. Rather than simply watching what was done in class students could follow along and see the results on their screens. Because all students had access to these tools in their rooms, in theory they could repeat classroom demonstrations or explore further on their own.

What does a student do who does not have a TI-89 or an expensive computer algebra system such as *Mathcad*? Or what about the student who does have access to these resources but cannot remember the keystrokes or correct syntax used in classroom demonstrations? Does he/she wait until the next class? More often than not, these procedural and syntactical stumbling blocks, and lack of resources become barriers to studying and self-motivated learning. Given that more than 50% of college students will be able to access the Internet from their dorm rooms and 80% or more will have access from some campus location(s) [1, p. 1], a viable solution may be applets.

### JAVA APPLETS

Our applets are written in Java and executed in a web browser. Why applets? Instructors can focus on concepts, modeling, and problem solving instead of teaching the syntax of *MathCad* or the keystrokes of the TI needed for procedural computations. Since there are no syntax or keystrokes to learn students start using and learning immediately. Moreover, learning concepts or lesson objectives are not by-products of having mastered certain software. In discussing this with students, one of the recurring complaints is the large amount of time required to become familiar with the software as opposed to understanding concepts. Students also say that they are more likely to use applets outside of the classroom since they do not experience the difficulties of having to learn a new program or procedure.

Java is: "A simple, object-oriented, network-savvy, interpreted, robust, secure, architecture neutral, portable, high-performance, multithreaded, dynamic language." [3, p. 1] In the education arena instructors can capitalize on the portable and dynamic qualities of the language. Unlike the TI-89 and MathCad, Java applets are not tied to a specific software package or computational device. As long as students have access to the Internet/Intranet, and a Java-compliant Web browser such as Netscape Navigator or Microsoft Internet Explorer they can use applets. Whether students have a Windows, Macintosh, Unix or other operating system, they can access and use the same applets demonstrated in the classroom.

Since applets can be very dynamic; rather than discussing what would happen as a parameter changes, instructors can show what happens instantaneously. The Graphical User Interface (GUI) of each applet can be designed so that users can easily modify parameters and/or variables and immediately see the results. This ease of use enables students to confidently perform "what if" analysis.



### CLASSROOM DEMONSTRATIONS

Two of the applets used with great success were: "The Army vs. Navy: The Long Trip" and "Harmonic Oscillator". Both are dynamic, interactive, computational, and concept illustrating. [4, p. 7] These applets offer numerous benefits that cannot be achieved through traditional, pedagogical methods. Course texts, transparencies, and blackboards are good resources, but they are not dynamic. These applets manipulate several parameters so the student can instantly see both the numerical and graphical solutions. The interactive features coupled with dynamic visualizations significantly assist students in understanding objectives and concepts.

#### Example 1: Army vs. Navy: The Long Trip

This applet was developed based on a situation that relates to all students at West Point.

*Each year hundreds of cadets depart the United States Military Academy enroute to the Army vs. Navy game. In the late 90's upon arriving at their hotel in Philadelphia a group of plebes (freshmen) realized that they had forgotten their long overcoats, a part of the next day's required uniform. To pass the time on the long drive back they developed a model for the distance away from the hotel as a function of time (distance from hotel in thirty minute time periods =  $-4.9978t^2 + 49.372t$ ).*

Therefore, at West Point, it has noteworthy advantages over other applets because little time is needed to explain the usefulness of the material or give a "real-world" application. Lessons that focus on the slope of a tangent line as the limiting value of secant line slopes, and instantaneous velocity as the limiting value of average rates of change are excellent venues for this applet. Our text, *Calculus: Concepts and Contexts* [5, pp. 142-148], does an outstanding job of presenting this material; however, to refer students to the examples in the text or construct drawings during class does not have the same impact as using an applet. The dynamic nature of this applet allows students to see real-time what the instructor or text describes or attempts to show via many static visual aids. Stewart uses six drawings in an attempt to drive home this idea, and that still may not be enough.

The dynamic nature and point-and-click capability of the GUI eliminate the need for countless static diagrams and/or long word descriptions. A great demonstration or exercise to illustrate that the limiting value of the slope of secant lines is the slope of the tangent line was achieved with the Army vs. Navy applet. Some of the requirements and questions posed to students were:

1. Set  $t_1$  at 3.5,  $t_0$  at 0.5, and use the applet to generate a numerical solution for the slope of the secant lines as  $t_0$  is increased in steps of 0.5 to 3.5.
2. Describe what happens to the slope of the secant lines as  $t_0$  increases to  $t_1$ ?
3. What does this mean?
4. An in-class demonstration begins with setting  $t_1$  at 3.5 and  $t_0$  at 0.5 as shown in Figure 1. We point out that the applet provides the average velocity between  $t_1$  and  $t_0$ , the slope of the secant line, and the instantaneous velocity at  $t_1$ , the slope of the tangent line.

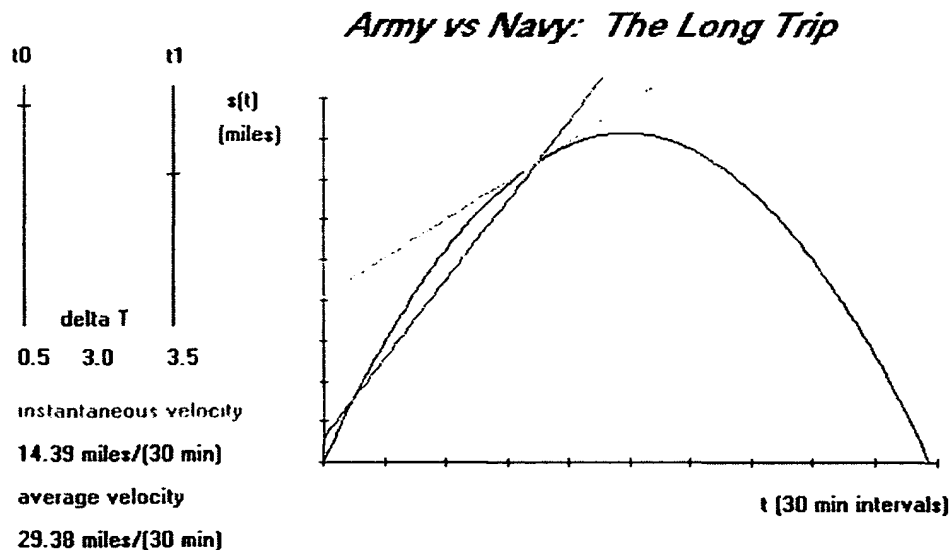


Figure 1: The Army vs. Navy applet shows the difference between the average velocity and instantaneous velocity when  $t_1 = 3.5$  and  $t_0 = 0.5$ .

Generating data for a numerical solution is only a point-and-click away. Users simply select the  $t_0$  slider and increase its value. Figure 2 shows one of the many instant views with data for the average velocity.

Figure 3 shows the applet when  $t_0 = t_1$ . The slope of the secant line equals to the slope of the tangent line; the instantaneous velocity equals the average velocity.

The numerical solution coupled with immediate visualization and an opportunity for hands-on training is a complete package for presenting this material as well as an out-of-class resource. The in-class demonstration or exercise allows students to successfully watch and understand, and they can repeat it during out-of-class study via the internet. While it is possible for students to accomplish something similar with *MathCad*, they must create a graph, *MathCad* sheet, and manually change parameter values.

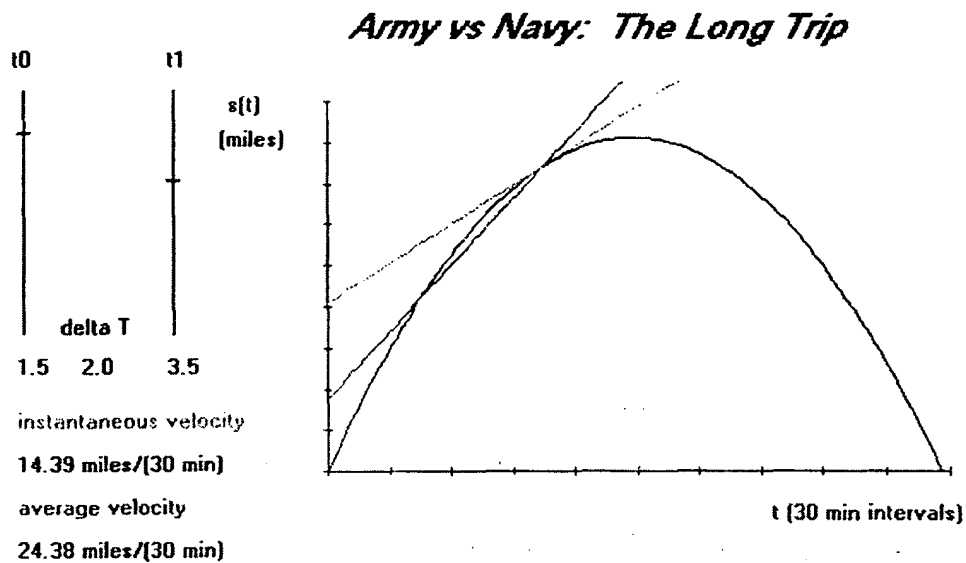


Figure 2: One of numerous real-time pictures a users sees as the value of  $t_0$  is increased to  $t_1$ .

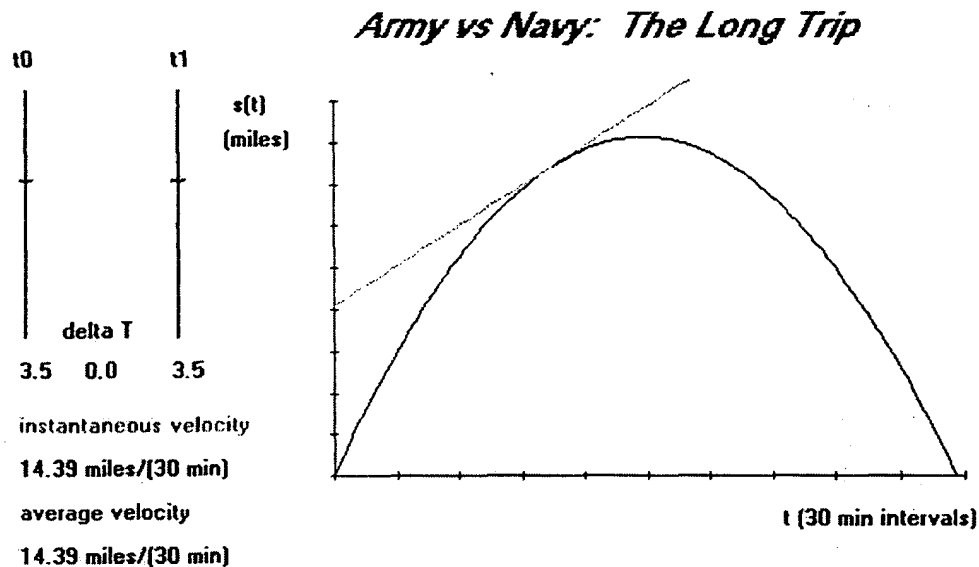


Figure 3: The Army vs. Navy applet illustrates that the limit of the average velocities is the instantaneous velocity when  $t_1$  and  $t_0$  equal 3.5.

### Example 2: Harmonic Oscillator

The harmonic oscillator is introduced in our Calculus and Introduction to Differential Equations course. Students become familiar with modeling harmonic motion and finding the solution to the second-order

differential equation that models this phenomenon. Explaining the qualitative behavior of the different classifications of oscillators is somewhat difficult without a vibrant, real-time graphing capability. We have found that the “Harmonic Oscillator” is a valuable tool for presenting the graphical solution as well as the relationship between roots of the characteristic equation, eigenvalues, and classification of the oscillator. *Mathcad* and the TI, though outstanding, cannot provide this level of user interaction and instantaneous visualization.

Lessons on harmonic oscillators require an understanding of the classification, relationship between classification and eigenvalues, and the ability to describe the long-term behavior (LTB) of the harmonic oscillator. Below is an excerpt from an out-of-class exercise. The accompanying table shows differential equations with initial conditions, initial value problems (IVP), that model harmonic oscillators. Complete the following requirements.

- a. Solve each of the IVPs listed in Table 1.
- b. Classify each of the oscillators.
- c. List the eigenvalues.
- d. Describe the LTB.
- e. What relationships do you conjecture between classification, eigenvalues, and LTB?
- f. Use the “Harmonic Oscillator” to check your work.

IVP $my'' + by' + ky = 0, \quad v(0) = v_0, \quad y(0) = y_0$	Classification	Eigenvalues $\lambda_1$ and $\lambda_2$	LTB
$y'' + 5y = 0, \quad v(0) = 4, \quad y(0) = 4$			
$y'' + y' + 5y = 0, \quad v(0) = 4, \quad y(0) = 4$			
$y'' + 6y' + 8y = 0, \quad v(0) = 4, \quad y(0) = 4$			
$y'' + 6y' + 9y = 0, \quad v(0) = 4, \quad y(0) = 4$			

Table 1: Students are asked to complete this table and then check their work using the applet.

Figures 4 and 5 show the applet when parameters for the first and second equation of Table 1 are used. Students “point and click” as needed to explore changing the parameters of the second-order equations. Figure 5 shows the effect of changing  $b$  from 0, as in Figure 4, to 1. With every click users are provided classification, eigenvalues, and a plot of the mass’ position.

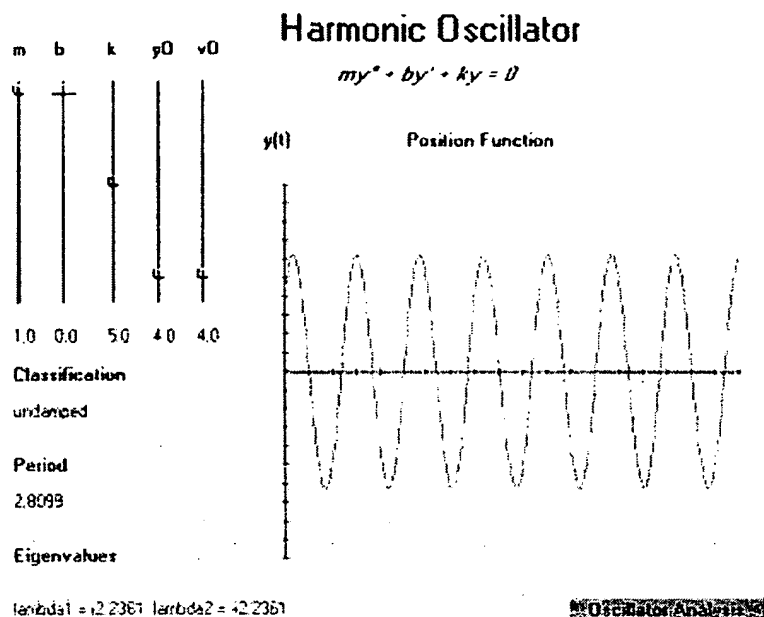


Figure 4: The "Harmonic Oscillator" applet with the parameters of the IVP,  $y'' + 5y = 0$ ,  $v(0) = 4$ ,  $y(0) = 4$ .

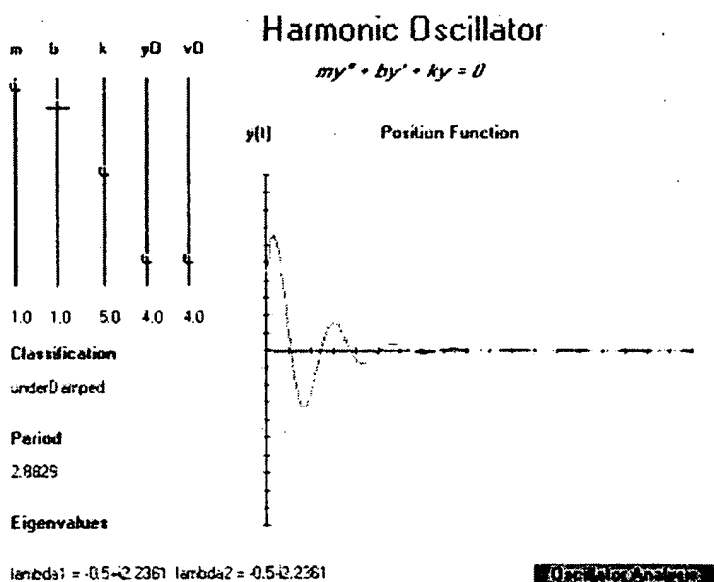


Figure 5: The "Harmonic Oscillator" applet with the parameters of the IVP,  $y'' + y' + 5y = 0$ ,  $v(0) = 4$ ,  $y(0) = 4$ .

This applet is exceptionally robust because it provides additional information to illustrate and reinforce concepts. Figure 6 shows how selecting the "Oscillator Analysis" button causes a window to appear that discusses the

qualitative behavior for the current set of parameters. This immediate feedback is provided on user demand via an error free process and does not require the investment in valuable study time to prepare a learning tool.

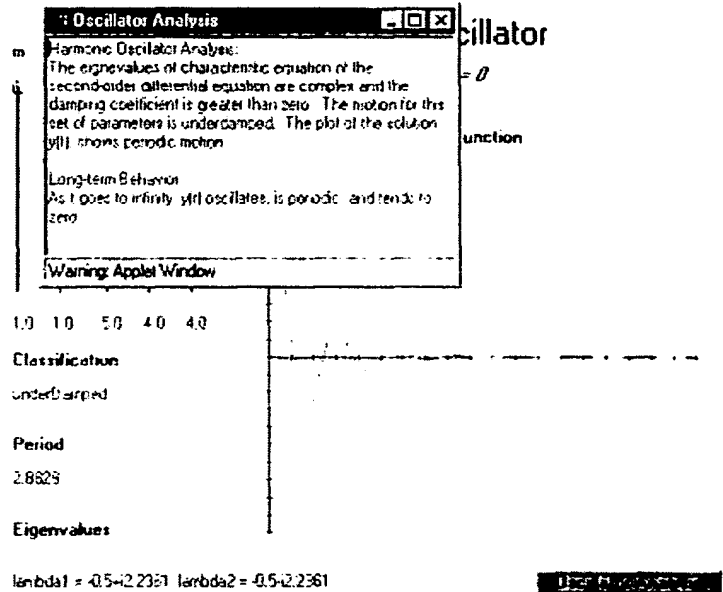


Figure 6: Students may select the "Oscillator Analysis" button for assistance in completing or verifying their answers to exercises.

A window appears that discusses the qualitative behavior of the oscillator.

## OUT-OF-CLASS STUDY

During out-of-class study students can access these and other applets to reinforce concepts covered in class. This also presents an opportunity for exploration and discovery. While the basic objectives and concepts are covered in class, using applets allows students to explore related concepts on their own without syntactical or procedural obstacles. This freedom to investigate fosters a deeper understanding of the material and more importantly aids students in learning how to learn. Whether in their room, a study partner's room, the library, or the department's mathematics clinic students can gain access over the internet.

## DEVELOPING THE TECHNOLOGY

A realistic and significant drawback can be the development of these tools. There are numerous applets available on-line; however, we felt few were suitable for our needs. Some did not illustrate objectives in our courses while others were merely animations and lacked user

interaction. We prefer applets to be both dynamic and interactive. There was also a lack of support for courses like Discrete Dynamical Systems so we chose to start from scratch. Developing, authoring, and packaging for deployment and presentation can be time-intensive and require significant human resources. Applets developed for our curriculum, including those discussed here, can be found at our department's web site at URL:

[www.dean.usma.edu/math/research/mathtech](http://www.dean.usma.edu/math/research/mathtech)

### BENEFIT REALIZED

The use of this technology is still relatively new and we have not conducted formal evaluations of its use in our classes. In-class surveys have been very promising. We feel that the opinions of students who have been instructed using these tools are a realistic measure of success. Students made the following comments:

*I think the applets helped a lot. This is especially true for the definition of the derivative. Visual explanations of why things work are very useful when trying to understand something that may not be clear.*

*I believe that the applets were the key to my understanding the derivative. The two different views allowed me to visualize the derivative and the graph of the derivative in relation to the graph of the function. It was also helpful to use when studying because I was able to substitute values into the fields to see the graphs of other functions.*

*I think the applet that showed the tangent line as the limit of the secant line was very helpful. I did not use them often but until I did use it I did not understand the concepts as well.*

*Applets give you a picture of what is going on so you can understand ... rather than just memorizing things.*

*Some people had trouble telling the difference between average velocity and instantaneous velocity without this applet [Army vs. Navy].*

*These [applets] were very useful. They were like computer aids that said in three pages what the book said in ten.*

*The applets allowed the instructor to show how something behaved visually and could follow up with examples quickly and answer questions with graphs.*

## CONCLUSION

The TI-89 and *MathCad* are truly outstanding; however, they cannot match the capabilities and possibilities available with Java applets. The dynamic, interactive and portability features of Java make it an ideal tool for the development of applets for classroom demonstrations and out-of-class resources. A limiting factor is the availability of human resources for authoring, developing, and packaging, or searching for appropriate material. Given the increasing emphasis on technology and interest to integrate numerous disciplines, this may be a limitation worth overcoming. Educators interested in developing applets may contact the author via e-mail for assistance. We can offer guidance about suitable references, tutorials, and Integrated Development Environments (IDE). The true measure of success may be the benefit realized by the students. We should take the necessary steps to eliminate obstacles to their learning. The use of Java applets is a step.

## REFERENCES

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